INVESTIGATE RULES OF GAMES

Practice 2 – Suggested solution (Case 2)

Objective of the task:

To investigate and generate rules for the probabilities of all possible differences between the two numbers if you select two random cards (from 2 sets of 4 consecutive numbers).

Solution (Case 2: the 2 sets of cards are different, none of the cards are the same and we draw 1 card from each set)

Level 1 -2:

I am going to use a table to find all possible outcomes when drawing two cards randomly (from 2 sets of 4 consecutive numbers, says: 1,2,3,4,5,6,7,8).

1st card / 2nd card	1	2	3	4
5	(5,1) 4	(5,2) 3	(5,3) 2	(5,4) 1
6	(6,1) 5	(6,2) 4	(6,3) 3	(6,4) 2
7	(7,1) 6	(7,2) 5	(7,3) 4	(7,4) 3
8	(8,1) 7	(8,2) 6	(8,3) 5	(8,4) 4
combination / difference				

I am going to do an experiment to find the experimental probabilities of each event.

Difference of the two cards drawn (from 2 set of 4 consecutive numbers)	**Tally**	**Frequency**	**Experimental Probability** $= \frac{\text{number of times the event occurred}}{\text{total number of trials}}$
1	𝍸 𝍸 ||	12	$\frac{12}{150} = 0.080$
2	𝍸 𝍸 𝍸 𝍸 |||	23	$\frac{23}{150} = 0.153$
3	𝍸 𝍸 𝍸 𝍸 𝍸 ||||	29	$\frac{29}{150} = 0.193$
4	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	35	$\frac{35}{150} = 0.233$
5	𝍸 𝍸 𝍸 𝍸 𝍸	25	$\frac{25}{150} = 0.167$
6	𝍸 𝍸 𝍸 |||	18	$\frac{18}{150} = 0.120$
7	𝍸 |||	8	$\frac{8}{150} = 0.053$
	Total:	150	1

I have drawn 2 cards randomly and find their differences for 150 times and recorded the above results.

Prediction-

According to the experimental probabilities, I predict that if I have done 300 times instead of 150, I will get around 24 trials to be "1" as the two cards' difference.

Level 3-4:

Suggest how these patterns happen to other similar case-

If the numbers of the cards get bigger, says: 2,3,4,5,6,7,8,9

I predict that the probabilities of the corresponding events will be the same, as I can see from the following table, the final difference is still 1, 2, 3, 4, 5, 6, 7. The rules or patterns of the probabilities are similar to case 1 as shown above.

1st card / 2nd card	2	3	4	5
6	(6,2) 4	(6,3) 3	(6,4) 2	(6,5) 1
7	(7,2) 5	(7,3) 4	(7,4) 3	(7,5) 2
8	(8,2) 6	(8,3) 5	(8,4) 4	(8,5) 3
9	(9,2) 7	(9,3) 6	(9,4) 5	(9,5) 4
combination / difference				

Level 5-6:
I am going to find and explain the rules and patterns from the experimental results.

From the above experimental results and probabilities, I find out that:

(i) There are 16 possible combinations of getting 2 cards and there are 7 different events for the differences of the number of the two cards, which is 1, 2, 3, 4, 5, 6, 7 in my case.
(ii) There is the most chance to get a 4, P (two cards' difference is 4), which is 0.233 from my experiment. It is like the peak.
(iii) Then the probabilities fall on both side of the peak until the smallest or the biggest differences, 1 and 7, which is 0.080 and 0.053 respectively from my experiment.
(iv) Getting P (two cards' difference is 4) will be 4 times of getting P (two cards' difference is 1)
(v) The total of the experimental probabilities of all the outcomes is 1.

Test some of the pattern-
I can test one of the above rule is correct by finding its theoretical probabilities.

Since there are 16 difference outcomes and only 2 of them is "6"
therefore the theoretical probability P (two cards' difference is 6) $=\frac{2}{16} = 0.125$
which is quite close the experimental probabilities 0.120

Level 7-8
Test/Justify my rules/patterns are correct by finding all of the theoretical probabilities (giving more examples)

Difference of the two cards drawn (from 2 set of 4 consecutive numbers)	**Theoretical Probability** $=\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$	**Experimental Probability** $=\frac{\text{number of times the event occurred}}{\text{total number of trials}}$
1	$\frac{1}{16} = 0.0625$	$\frac{12}{150} = 0.080$
2	$\frac{2}{16} = 0.1250$	$\frac{23}{150} = 0.153$
3	$\frac{3}{16} = 0.1875$	$\frac{29}{150} = 0.193$
4	$\frac{4}{16} = 0.2500$	$\frac{35}{150} = 0.233$
5	$\frac{3}{16} = 0.1875$	$\frac{25}{150} = 0.167$
6	$\frac{2}{16} = 0.1250$	$\frac{18}{150} = 0.120$
7	$\frac{1}{16} = 0.0625$	$\frac{8}{150} = 0.053$
Total:	1	1

By comparing the experimental probabilities and the theoretical probabilities:

(i) There are 16 possible combinations of getting 2 cards and there are 7different events for the differences of the number of the two cards, which is 1, 2, 3, 4, 5, 6, 7 in my case (see table under level 1-2.
(ii) There is the most chance to get a 4, P (two cards' difference is 4), which is 0.25 (theoretically) and close
(iii) to 0.233 from my experiment. It is the peak.
(iv) Then the probabilities fall on both side of the peak until the smallest or the biggest differences, 1 and 7, which is 0.0625. When compare with the experimental results 0.080 and 0.053 respectively are still close.
(v) Getting P (two cards' difference is 4) is 0.25 which is 4 times of getting P (two cards' difference is 1) 0.0625 (since $0.25 \div 0.0625 = 4$)
(vi) The total of the experimental probabilities of all the outcomes is 1 (from the table).

Conclusions:

After going through the whole process of investigation and justification by finding the experimental and theoretical probabilities, I conclude that my rules and patterns for the probabilities of the difference by drawing 2 cards randomly from 2 sets of 4 consecutive cards are correct.

(i) There are 16 possible combinations if the two sets of card are the same, and the seven differences are 1, 2, 3, 4, 5, 6, 7,

(ii) While the middle one (difference of the two cards is 4) will have the greatest probability and,

(iii) Then probabilities fall on both side of 4, until the two extreme (difference of the two cards is 1 and 7) have the smallest probability,

(iv) Getting P (two cards' difference is 4) is 4 times of getting P (two cards' difference is 1),

(v) The total of the experimental probabilities of all the outcomes is 1.